Update of the $e^+e^- \to \pi^+\pi^-$ cross section measured by SND detector in the energy region $400 < \sqrt{s} < 1000$ MeV.

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The corrected cross section of the $e^+e^- \to \pi^+\pi^-$ process measured in the SND experiment at the VEPP-2M e^+e^- collider is presented. The update is necessary due to a flaw in the $e^+e^- \to \pi^+\pi^-$ and $e^+e^- \to \mu^+\mu^-$ Monte Carlo events generators used previously in data analysis.

The spherical neutral detector SND [1] operated from 1995 to 2000 at VEPP-2M e^+e^- collider [2]. One of the recent SND results was the measurement of the $e^+e^- \to \pi^+\pi^-$ process cross section in the energy region $\sqrt{s} < 1000$ MeV [3]. The systematic error of the cross section determination was estimated to be 1.3 %. Studies of the $e^+e^- \to \pi^+\pi^-$ reaction allow us to determine the ρ and ω meson parameters and provide information on the G-parity violation mechanism in the $\omega \to \pi^+\pi^-$ decay.

In the last time the comprehension of the high precision results of the muon anomalous magnetic moment measurement [4, 5] attracted heightened attention to the $e^+e^- \to \pi^+\pi^-$ cross section.

The comparison of the $e^+e^- \to \pi^+\pi^-$ process cross section with the spectral function in the $\tau^\pm \to \pi^\pm\pi^0\nu_\tau$ decay [6, 7, 8] is used to test the conservation of the vector current (CVC).

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Theoretical calculations of the $e^+e^- \rightarrow e^+e^-, \pi^+\pi^-, \mu^+\mu^-$ reactions cross sections play important role in the $e^+e^- \rightarrow \pi^+\pi^-$ process measurements. They are necessary for luminosity measurements ($e^+e^- \rightarrow e^+e^-$ events), for the $e^+e^- \rightarrow \mu^+\mu^-$ background subtraction, for the radiative corrections and $e^+e^- \rightarrow \pi^+\pi^-$ detection efficiency determination. The $e^+e^- \rightarrow \pi^+\pi^-, \mu^+\mu^-$ cross sections were calculated according to the formulae of Ref. [9, 10], which take into account the photons radiation by the initial and final state particles and have accuracy of about 0.2%.

Recently it was found that the $e^+e^- \rightarrow \pi^+\pi^-$ and $\mu^+\mu^-$ Monte Carlo events generators used in the SND data analysis were not quite correct and understated the $e^+e^- \rightarrow \pi^+\pi^-$ and $e^+e^- \rightarrow \mu^+\mu^-$ cross sections by about 2.5% and 1.5% respectively.

In this paper, in order to correct the error, the measured cross section σ_0 is multiplied by correction factors:

$$\sigma = \sigma_0 \cdot \delta_\pi \cdot \delta_\mu, \tag{1}$$

where δ_π and δ_μ are corrections due to mistakes in the $e^+e^- \rightarrow \pi^+\pi^-$ and $e^+e^- \rightarrow \mu^+\mu^-$ cross sections calculations respectively. The δ_π and δ_μ coefficients were determined using the MCGPJ $e^+e^- \rightarrow \mu^+\mu^-$, $\pi^+\pi^-$ events generator [11], which is based on the same approach [9, 10]. The applied corrections have not altered the systematic error value of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section measurement, which is 1.3 % for the energy region $\sqrt{s} \geq 420$ MeV and 3.2 % for $\sqrt{s} < 420$ MeV.

The energy dependence of the δ_π and δ_μ is shown in Fig.1. The error in the $e^+e^- \rightarrow \mu^+\mu^-$ cross section calculation is significant only in the energy region $\sqrt{s} < 500$ MeV. The corrected values of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section $\sigma_{\pi\pi}(s)$, of the form factor

$$|F_\pi(s)|^2 = \frac{3s}{\pi\alpha^2\beta^3}\sigma_{\pi\pi}(s), \ \ \beta = \sqrt{1 - 4m_\pi^2/s} \tag{2}$$

and of the bare cross section (the cross section without vacuum polarization contribution but with the final state radiation taken into account) are listed in Table I. The cross section decreased by two systematic errors in average. The results presented in Table I supersede the results quoted in the original work [3] Table 1.

The comparison of the obtained cross section with CMD-2 [12] and KLOE [13] measurements is shown in Figs.2,3. The CMD-2 result exceeds the SND data by 1.4±1.5% in average. Here the error includes both systematic and statistical uncertainties. The uncorrected SND

cross section exceeds the CMD-2 one by the same value. In the KLOE experiment at the DAΦNE ϕ-factory the form factor $|F_\pi(s)|^2$ was measured using "radiative return" method with the systematic error of 0.9 % [13]. In Ref.[13] the bare form factor values are listed. So in order to compare the KLOE result with the SND one, the form factor was appropriately dressed by us. The results of this comparison are shown in Fig.3. The difference between SND and KLOE data is energy dependent. The point that jumped out is situated in the region of the sharp rise of the cross section due to the $\rho - \omega$ interference. The KLOE measurement is in conflict with the SND result as well as with the CMD-2 one.

The cross section was fitted as described in the original work [3]. The fit results together with their deviation from the previous outcomes [3] (in units of measurement errors) are listed in Table II. These values supersede the results of the previous work [3]. All parameters except $\sigma(\rho \to \pi^+\pi^-)$, $B(\rho \to e^+e^-) \times B(\rho \to \pi^+\pi^-)$ and $\Gamma(\rho \to e^+e^-)$ changed by less than 0.5 error values, while $\sigma(\rho \to \pi^+\pi^-)$, $B(\rho \to e^+e^-) \times B(\rho \to \pi^+\pi^-)$ and $\Gamma(\rho \to e^+e^-)$ – by less than two errors. The discussion of the parameters and conclusions made in the original work [3] are still valid.

The comparison of the $e^+e^- \to \pi^+\pi^-$ cross section obtained under the CVC hypothesis from the τ-lepton spectral function from the $\tau^- \to \pi^-\pi^0\nu_\tau$ decay [7, 8] with the isovector part of the cross section measured by SND is shown in Fig.4, 5. In order to compare with the τ spectral function, the radiative correction $S_{EW} = 1.0198 \pm 0.0006$ [7, 8, 14] was applied. The $e^+e^- \to \pi^+\pi^-$ cross section was undressed from the vacuum polarization, the contribution from the $\omega \to \pi^+\pi^-$ decay was excluded and correction for the $\pi^\pm$ and π^0 mass difference was applied. As a result one can see the picture well known from Ref.[8, 15]. It is interesting that the difference between e^+e^- and τ data is approximately equal to the value of the accepted vacuum polarization contribution to the e^+e^- annihilation. The comparison of τ data with dressed $e^+e^- \to \pi^+\pi^-$ cross section is shown in Fig.6, 7.

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TABLE I: The results of the $e^+e^- \to \pi^+\pi^-$ cross section measurements. $\sigma_{\pi\pi}$ and $|F_\pi|^2$ are the cross section and the form factor of the $e^+e^- \to \pi^+\pi^-$ process, $\sigma_{\pi\pi}^{pol}$ is the $e^+e^- \to \pi^+\pi^-$ undressed cross section without vacuum polarization but with the final state radiation. Only uncorrelated errors are shown. The correlated systematic error σ_{sys} is 1.3 % for $\sqrt{s} \geq 420$ MeV and 3.2 % for $\sqrt{s} < 420$ MeV.

$\sqrt{s}$ (MeV)	$\sigma_{\pi\pi}$(nb)	$|F_\pi|^2$	$\sigma_{\pi\pi}^{pol}$(nb)
970.0	76.68± 1.79	3.78±0.09	75.06± 1.75
958.0	91.33± 1.96	4.41±0.09	89.22± 1.91
950.0	101.52± 1.93	4.83±0.09	99.07± 1.88
940.0	115.14± 1.57	5.38±0.07	112.25± 1.53
920.0	147.78± 5.15	6.66±0.23	143.57± 5.00
880.0	246.38± 2.80	10.30±0.12	237.93± 2.70
840.0	450.70± 4.19	17.46±0.16	433.39± 4.03
820.0	622.54± 5.54	23.19±0.21	597.24± 5.31
810.0	715.94± 6.21	26.15±0.23	685.26± 5.94
800.0	822.66± 7.05	29.46±0.25	785.42± 6.73
794.0	859.35± 7.19	30.41±0.25	815.84± 6.83
790.0	855.55±16.98	30.04±0.60	806.96±16.02
786.0	874.23± 7.42	30.45±0.26	820.44± 6.96
785.0	887.68± 8.81	30.86±0.31	835.25± 8.29
784.0	940.42±19.12	32.62±0.66	890.94±18.11
783.0	1022.45±10.99	35.40±0.38	979.52±10.53
782.0	1106.69±26.44	38.24±0.91	1073.85±25.66
781.0	1161.62±10.84	40.06±0.37	1138.88±10.63
780.0	1233.58±10.17	42.45±0.35	1220.00±10.06
778.0	1314.33± 9.78	45.05±0.34	1309.00± 9.74
774.0	1331.59± 9.87	45.28±0.34	1326.86± 9.83
770.0	1302.16± 9.67	43.92±0.33	1296.23± 9.63
764.0	1304.40± 9.80	43.47±0.33	1297.32± 9.75
760.0	1308.40±10.08	43.26±0.33	1301.28±10.03
750.0	1291.96±22.80	41.86±0.74	1288.31±22.74
720.0	1060.14± 7.11	32.31±0.22	1064.32± 7.14

Table I: (Continued)

$\sqrt{s}$ (MeV)	$\sigma_{\pi\pi}$(nb)	$|F_\pi|^2$	$\sigma_{\pi\pi}^{pol}$(nb)
690.0	764.53± 8.31	21.92±0.24	769.56± 8.36
660.0	543.75± 6.24	14.66±0.17	546.05± 6.27
630.0	398.61± 8.73	10.11±0.22	399.49± 8.75
600.0	296.06±10.92	7.08±0.26	296.17±10.92
580.0	261.49±14.78	6.01±0.34	261.11±14.76
560.0	230.91±12.69	5.12±0.28	230.54±12.67
550.0	221.00±17.83	4.81±0.39	220.33±17.78
540.0	215.61±13.79	4.62±0.30	214.99±13.75
530.0	202.32±23.04	4.26±0.49	201.77±22.98
520.0	179.55±10.42	3.72±0.22	179.10±10.39
510.0	175.37±16.81	3.58±0.34	174.62±16.74
500.0	176.32±10.93	3.55±0.22	175.60±10.89
480.0	165.60± 9.72	3.26±0.19	165.01± 9.69
470.0	143.61±13.28	2.81±0.26	143.14±13.24
450.0	140.47±14.24	2.71±0.28	139.82±14.17
440.0	114.75±15.51	2.22±0.30	114.26±15.44
430.0	109.25±12.54	2.11±0.24	108.83±12.49
410.0	125.06±18.92	2.46±0.37	124.70±18.87
390.0	116.37±21.78	2.39±0.45	116.17±21.74

TABLE II: The ρ and ω mesons parameters measured in this work. In the third column the parameters deviations from the original work results [3] in units of measurement errors are listed.

Parameters	Values	Deviations
m_ρ, MeV	774.6±0.4±0.5	0.5
Γ_ρ, MeV	146.1±0.8±1.5	0.2
$\sigma(\rho \to \pi^+\pi^-)$, nb	1193±7±16	1.6
$B(\rho \to e^+e^-) \times B(\rho \to \pi^+\pi^-)$	$(4.876 \pm 0.023 \pm 0.064) \times 10^{-5}$	1.6
$\Gamma(\rho \to e^+e^-)$, keV	$7.12 \pm 0.02 \pm 0.11$	1.7
$\sigma(\omega \to \pi^+\pi^-)$, nb	29.3±1.4±1.0	0.3
$B(\omega \to e^+e^-) \times B(\omega \to \pi^+\pi^-)$	$(1.225 \pm 0.058 \pm 0.041) \times 10^{-6}$	0.3
$\phi_{\rho\omega}$, degrees	113.7±1.3±2.0	0.1
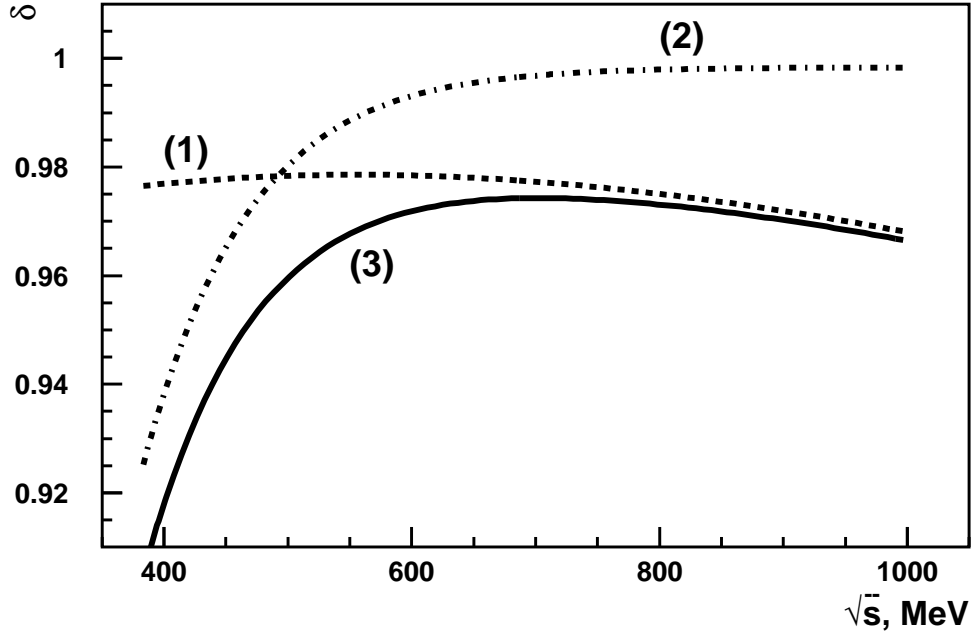
FIG. 1: Corrections $\delta = \delta_\pi \cdot \delta_\mu$ to the $e^+e^- \to \pi^+\pi^-$ cross section [3], which take into account the mistakes of the $e^+e^- \to \pi^+\pi^-$ (1) and $e^+e^- \to \mu^+\mu^-$ (2) cross sections calculations and the total correction δ (3).
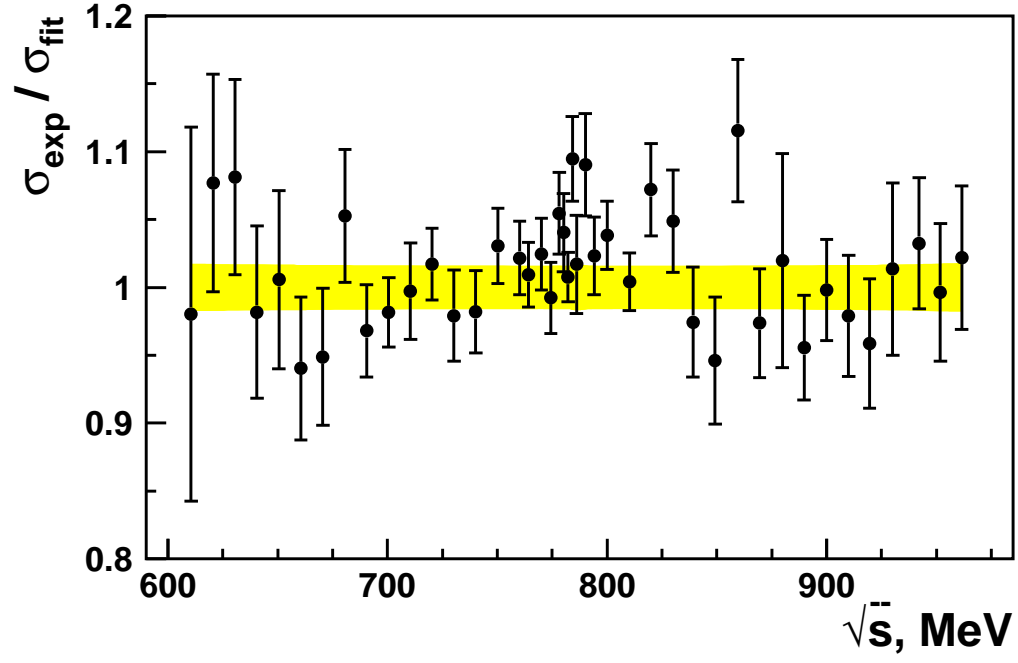
FIG. 2: The ratio $\sigma_{exp}/\sigma_{fit}$ of the $e^+e^- \to \pi^+\pi^-$ cross section measured by CMD-2 [12] to the SND fit curve. The shaded area shows the joint systematic error.

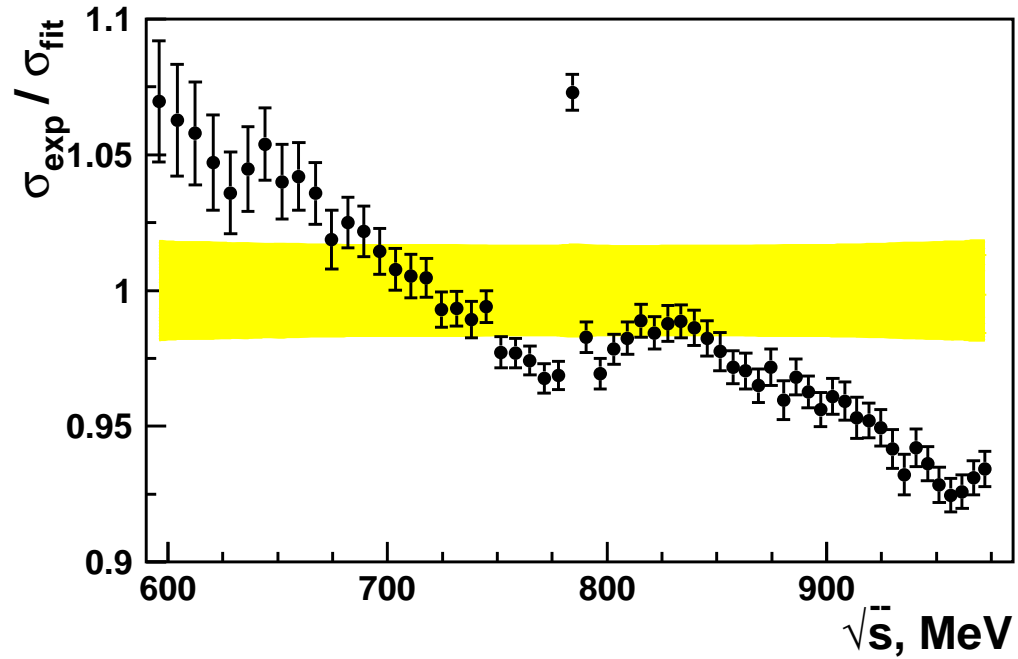FIG. 3: The ratio $\sigma_{exp}/\sigma_{fit}$ of the $e^+e^- \to \pi^+\pi^-$ cross section measured by KLOE [13] to the SND fit curve. The shaded area shows the joint systematic error.

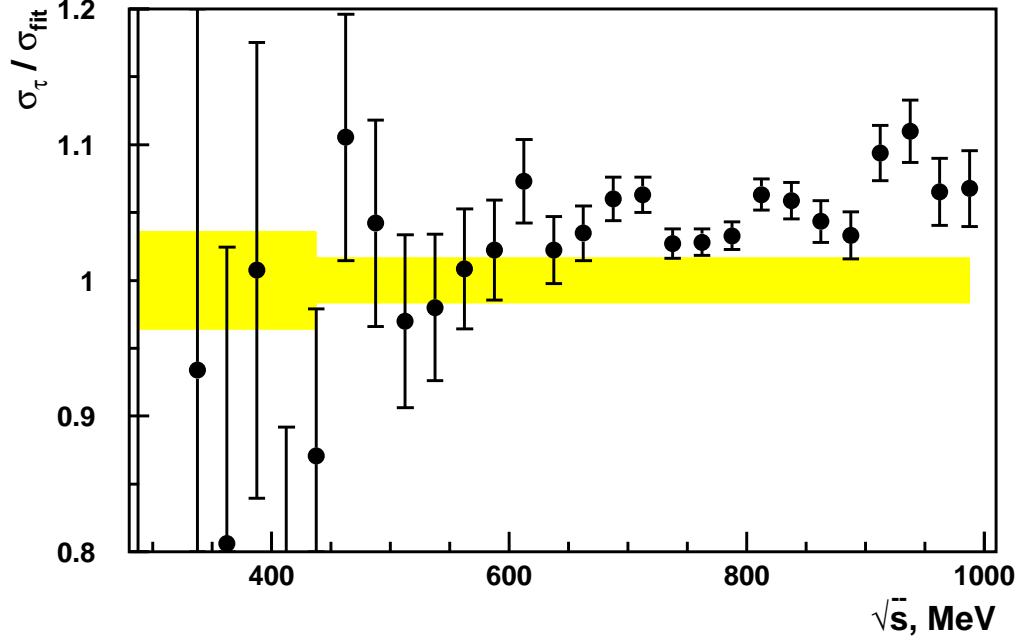FIG. 4: The ratio of the σ_τ/σ_{fit} of the $e^+e^- \to \pi^+\pi^-$ cross section calculated from the $\tau^- \to \pi^-\pi^0\nu_\tau$ decay spectral function measured by CLEOII [7] to the isovector part of the $e^+e^- \to \pi^+\pi^-$ cross section corrected in this work. The shaded area shows the joint systematic error.
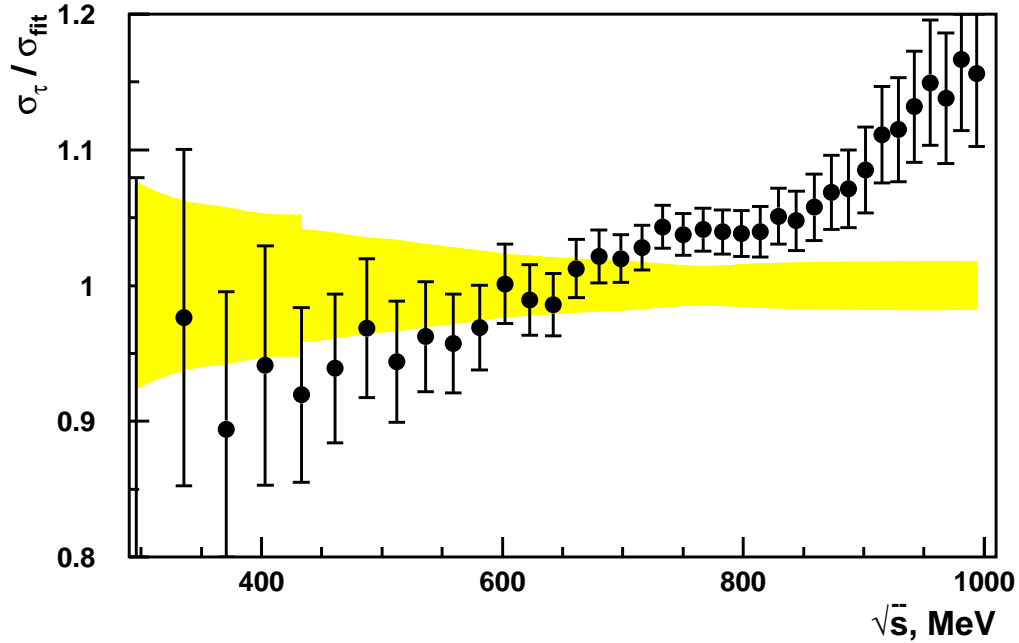

FIG. 5: The ratio of the σ_τ/σ_{fit} of the $e^+e^- \to \pi^+\pi^-$ cross section calculated from the $\tau^- \to \pi^-\pi^0\nu_\tau$ decay spectral function measured by ALEPH [8] to the isovector part of the $e^+e^- \to \pi^+\pi^-$
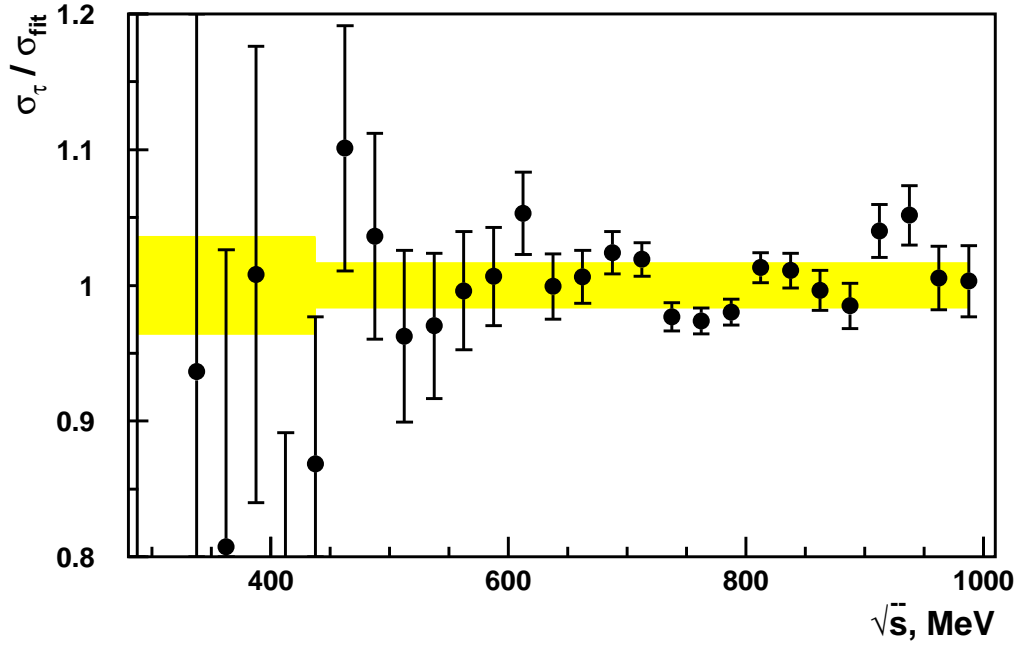
FIG. 6: The ratio of the σ_τ/σ_{fit} of the $e^+e^- \to \pi^+\pi^-$ cross section calculated from the $\tau^- \to \pi^-\pi^0\nu_\tau$ decay spectral function measured by CLEOII [7] to the isovector part of the $e^+e^- \to \pi^+\pi^-$ cross section corrected in this work when the vacuum polarization contribution is not extracted from the SND data. The shaded area shows the joint systematic error.
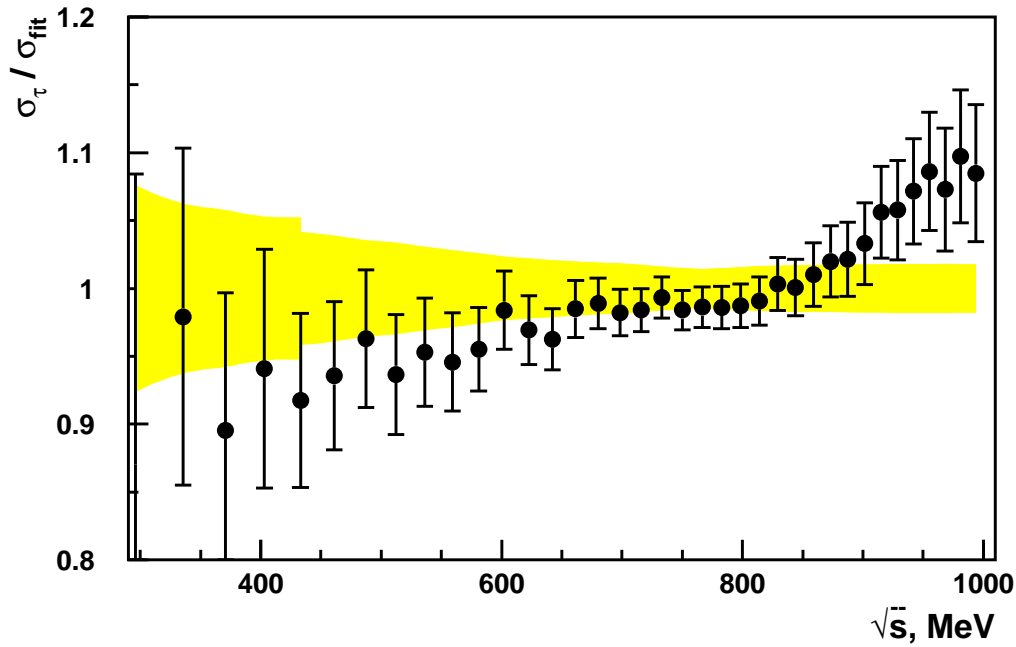

FIG. 7: The ratio of the σ_τ/σ_{fit} of the $e^+e^- \to \pi^+\pi^-$ cross section calculated from the $\tau^-\pi^-\pi^0\nu_\tau$ decays spectral function measured by ALEPH [8] to the isovector part of the $e^+e^- \to \pi^+\pi^-$